OPTIMAL POWER CONTROL LAW FOR MULTI-MEDIA
MULTI-RATE CDMA SYSTEMS

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Abstract — This paper considers the optimal reverse-link power control law for multi-media multi-rate DS/CDMA systems which offer services with different transmission rates (VBR or CBR) and quality of service (Qos) requirements. In the multi-media multi-rate CDMA system, the transmission power of a mobile station (MS) is proportional to a power control law, which is a function of its distance to the base station (BS), the transmission rates and interference level (number of active users). Each transmitted signal from MS will be affected by path loss, Rayleigh fading and log-normal shadowing in the reverse link. Bit error probability of the asynchronous multi-rate CDMA system that takes MS locations into account is derived first in order to derive the optimum power control functions. By applying the optimal power control functions to mobile stations, the Qos (bit error probability) requirements can be met, the system capacity (number of simultaneously transmitted users) can be maximized, and power consumptions can be minimized. Since the difficulty of deriving the optimal power control law in closed form, we discretize the power control function so that the dynamic programming (DP) can be utilized to obtain the optimal solution. The numerical results show that the optimal power control law can increase system capacity and decrease power consumption compared with other power control laws (eg. equal received bit energy) in the multi-media multi-rate CDMA system.

I. INTRODUCTION

DS/CDMA has been proposed to be one of the multiple access techniques for future personal communication systems (PCS) [1]. The technology is attractive because of the possibility of achieving higher capacity and conveying media with different transmission rate. Since PCS promises to support multi-media traffic such as voice(audio), data, video, fax etc., multi-media multi-rate CDMA is getting more important in the wireless communication. Due to the near-far effect and shadowing in CDMA environment, mobile stations in disadvantage situation (far from base station or being shadowed by buildings) may suffer from poor signal quality. To mitigate the near-far and shadowing effect on system, power control that maintain required signal quality is necessary for forward and reverse link.

Many researches on CDMA power control law have been presented for single rate single media system [2, 3, 5]. Basically in these systems, the power control law for mobiles is proportional to the distance to base station and interference level so that the received signal power for each mobile is maintained at a minimum level to meet the bit error rate requirement. However, those power control law may be inappropriate for multi-media multi-rate CDMA systems in fading environment where different services have different transmission rate (CBR or VBR) and different quality of service (Qos) requirements.

For PCS system designers, system capacity and power resources are the major concerns of a CDMA system, so power control not only needs to control the near-far effect but also to optimize the system capacity and other resource with target QoS requirements. In this paper, we investigate and present optimal power control law which maximize system capacity and minimize power consumption for the multi-media multi-rate CDMA systems. In the multi-rate CDMA, the transmission power of a MS is proportional to a power control law, which is a function of its distance to the BS, transmission rates and interference level (number of active users). Each transmitted signal from MS will be affected by path loss, Rayleigh fading and log-normal shadowing in the reverse link. Rayleigh fading and log-normal shadowing has been used to model mobile radio channel by many researchers [4, 10]. The model is not only practically reasonable but also mathematical tractable. Since in this paper Qos requirements are measured in terms of bit error rate (BER), the BER of the coherent 1 asynchronous binary multi-rate DS/CDMA including power control functions are derived. Convolutional coding with Viterbi decoding is employed so that the performance can be improved. The conventional Gaussian approximation and enhanced Gaussian approximation are used in the paper. From the BER expressions, we need to derive the optimal power control functions that optimize the objectives. However, it is difficult to obtain a closed form solution, thus we utilize the dynamic programming method to get the discrete power control function. From the numerical results, optimal power control law increase the system capacity and decrease power consumption for multi-media multi-rate CDMA systems compared to other power control law (eg. equal received bit energy).

II. SYSTEM DESCRIPTION OF MULTI-RATE DS/CDMA

The DS/CDMA model we discuss here is a single-cell system where information is transmitted in one of the $S$ rates and for each rate $R_i$, $i \in [0, S - 1]$, there are $K_i$ active mobiles. Assuming the same fixed bandwidth $W$ (the same chip duration $T_c = \frac{1}{W}$) are allocated to every user, $R_0 \leq R_1 \leq \cdots \leq R_{S-1}$ (thus for bit duration $T_b$, $T_b \geq T_0 \geq \cdots \geq T_{S-1}$ and processing gain $G_i = \frac{W}{R_i} = \frac{W}{P_i}$, $G_0 \geq G_1 \geq \cdots \geq G_{S-1}$). All users are randomly distributed in the cell with probability density function

$$f_r(r) = 2\pi r, \quad 0 \leq r \leq 1$$

where $r$ is the distance from the MS to the BS. The cell radius is normalized to 1 without loss of generality. For asynchronous

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1 In real world, we usually use noncoherent demodulation in the reverse link.
binary DS/CDMA, the transmitted signal of user \( k \) with bit duration \( T_b \) can be expressed as
\[
s_{ik}(t) = \sqrt{2P_{ik}} b_{ik}(t) a_{ik}(t) \cos(2\pi f_c t + \theta_{ik})
\]
for \( 0 \leq k \leq K_i \) and \( 0 \leq i \leq S - 1 \). In (2), \( P_{ik} \) is the transmitted signal power which is expressed by the power control function \( g_i(t, \tau_k) \in [0, 1] \), a function of the distance from the mobile to base station.

\[
P_{ik} = g_i(t, \tau_k) P_b
\]
where \( P_b \) is the maximum transmission power. The data waveform \( b_{ik}(t) \) consists of a sequence of mutually independent rectangular pulses \( b_{ik} \) of duration \( T_i \) and amplitude taking values \(+1\) or \(-1\) with equal probability. The chip waveform \( a_{ik}(t) \) consists of a sequence of rectangular pulses \( a_{ik} \) of duration \( T_c \) and amplitude \(+1\) or \(-1\). In practical application, the spreading functions for every user is usually deterministic and selected carefully to decrease the cross-correlation between different sequences. In the paper, we assume that the chip sequences are randomly generated. The term \( f_c \) is the carrier frequency and \( \theta_{ik} \) is the phase introduced by the \( k \)th modulator with bit duration \( T_i \).

### A. Channel Model

Each transmitted signal from MS to BS is affected by path loss, Rayleigh fading and log-normal shadowing. Path loss is determined by the distance between the MS and the BS, which is proportional to \( r^{-n} \), where \( n \) is the propagation exponent depending on environment (\( n = 4 \) is a typical value in land mobile radio environment [2]). Rayleigh fading and log-normal shadowing are mutually independent and assumed to be invariant during the data bit duration. The output signal of the channel is further corrupted by the additive white Gaussian noise \( n(t) \) of two-sided spectral density \( N_0/2 \). Based on the above assumptions, the received signal at base station is given by

\[
y(t) = \sum_{i=1}^{S-1} \sum_{k=0}^{K_i-1} s_{ik}(t - \tau_k) \alpha_{ik} e^{j2\pi f_c T_i t} + n(t)
\]
where \( \tau_k \) is the relative delay \( 0 \leq \tau_k \leq T_i \), \( \alpha_{ik} \) stands for Rayleigh fading with parameter \( \sigma_{\alpha}^2 \). \( \alpha_{ik} \) is a Gaussian random variable with mean 0 and variance \( \sigma_{\alpha}^2 \).

### B. Receiver Model

The receiver is assumed to be coherent and matched to the signal of user \( 0 \) of rate \( R_m \). Considering the reception of data bit \( b_{0,0} \), the output of the corresponding correlation receiver is

\[
Z_{m0} = N_{m0} + \sqrt{\frac{P_{m0}}{2}} \alpha_{m0} e^{j2\pi f_c T_{m0} b_{0,0}} + I_{m0}
\]

The random variable \( N_{m0} \) is Gaussian with mean 0 and variance \( N_0 T_{m0}/4 \). The random variable \( I_{m0} \) is given by

\[
I_{m0} = T_{m0}^{-1} [F_1 + F_2] \cos \phi_{m0}
\]
where \( \phi_{m0} = \theta_{m0} - 2\pi f_c T_{m0} \),

\[
F_1 = \sum_{k=1}^{K_i-1} \sqrt{\frac{P_{m0}}{P_{m0} \alpha_{m0} e^{j2\pi f_c T_{m0}} I_{m0}}} \alpha_{m0} e^{j2\pi f_c T_{m0}} B(m_0; 0; m, k; \tau_{mk})
\]

and

\[
F_2 = \sum_{i=0, i \neq m}^{S-1} \sum_{k=1}^{K_i-1} \sqrt{\frac{P_{m0}}{P_{m0} \alpha_{m0} e^{j2\pi f_c T_{m0}} I_{m0}}} \alpha_{m0} e^{j2\pi f_c T_{m0}} B(m_0; 0; i, k; \tau_{ik})
\]

In (7) and (8), the term \( B(m, l; i, k; \tau) \) can be expressed by the cross-correlation function,

\[
B(m, l; i, k; \tau) = \int_0^{T_m} b_{ik}(t - \tau) a_{ik}(t - \tau) a_{il}(t) dt
\]

For practical DS/CDMA application, since chip sequences are known (for example, m-sequence and Gold sequences), the values of the crosscorrelation functions can be readily computed. In this paper, random binary sequence are used and for different users the random sequence are mutually independent.

### C. BER: Gaussian Approximation

For binary asynchronous DS/CDMA and AWGN channel, Gaussian approximation [6] and characteristic function method [7] have been used to compute the bit error probability. In this section, Gaussian approximation for fading channel is used. We further assume that the random variables data waveform, phase angle, time delay are mutually independent each other and also are mutually independent for different users. By applying central limit theory (chip number \( G_i \) and number of users \( K_i \) are large enough), the bit error probability of the desired signal (from 0th user of rate \( R_m \)) can be obtained from the conditional bit error probability given the distance \( r_{m0} \), Rayleigh fading \( \gamma_{m0} \) and shadowing \( \beta_{m0} \).

\[
P_{m0}^{(m)}(r_{m0}) = E[Q(\gamma_{m0})] = E[Q(\gamma_{m0})]
\]

In (10) and (11), \( \alpha = \alpha/\sigma_{\alpha}^2 \) is the normalized Rayleigh fading and \( \beta = \beta/\sigma_{\beta}^2 \) is a shifted Gaussian random variable with mean \( -\alpha \) and variance \( \sigma_{\beta}^2 \). The term \( \Sigma_m \) is expressed by

\[
E[r_{m0,0}^n g_m(r_{m0})] = \frac{E[r_{m0,0}^n g_m(r_{m0})]}{E[r_{m0,0} g_m(r_{m0})]} + \frac{2}{3G_m r_{m0,0} g_m(r_{m0})}
\]

3 In Fact, \( \sigma_{\beta}^2 = 10^{10} \), where \( c \) is Gaussian distributed with zero mean and standard deviation \( \sigma_{c} \). Therefore \( \sigma_{\beta} = \frac{10^2}{10^{10}} \sigma_{c} \).
where $E_b^{(i)} r_{mk} g_i(r_{ik})$ is the average received bit energy to noise density ratio for user $k$ with rate $i$ and $E_b^{(i)}$ can be expressed as

$$E_b^{(i)} = P_i T_i E_a^{(i)} E_{\alpha^{(i)}}$$

$$= P_i T_i 2 \sigma_{\alpha^{(i)}} e^{-\sigma_{\alpha^{(i)}}^2 / 2} = T_i P_0^{(i)}$$ (13)

Let the maximum received bit energy per noise density be defined by

$$\frac{E_b^{(i)}}{N_0} = \frac{T_0 P_0^{(i)}}{N_0} = \frac{E_b^{(i)}}{N_0}$$

Thus the equation (12) can be written as

$$\sum_{k=1}^{K_m} (r_{mk}) = \left[ \frac{E_b^{(i)}}{N_0} \gamma_{m0} r_{m0} g_m(r_{m0}) \right]^{-1} + \frac{2}{3G_m r_{m0} g_m(r_{m0})}$$

$$\times \sum_{k=1}^{K_m} E_{\alpha^{(i)}} g_i(r_{ik}) (15)$$

**D. BER: Enhanced Gaussian Approximation**

Since in the multi-rate DS/CDMA system we assume there have $S$ sub-systems (different rate), some user's processing gain are very small. This will make central limit theory more loose to the accurate BER. In order to get an accurate BER we use an improved Gaussian approximation. The proposition is that if we condition all Rayleigh, log-normal random variable and distances, the Gaussian approximation converges to the actual probability density faster than taking average of all fading terms (11).

The average bit error rate of digital data of mobile $0$ (in rate $R_m$, at distance $r_{m0}$) received by the base station is given by

$$P_{b(m)}(r_{m0}) = E[Z_{m0}] \left[ \frac{E[Z_{m0}]}{\text{SNR}(\hat{a}, \hat{b}, r)} \right]$$

$$= E[Z_{m0}] \left[ \frac{E[Z_{m0}]}{\text{SNR}(\hat{a}, \hat{b}, r)} \right]$$ (16)

where vectors $a = \{ \alpha_{ik}, i \}, b = \{ \beta_{ik}, i \}, r = \{ r_{ik} \}$ and $\hat{a} = \{ \hat{\alpha}_{ik} \}, \hat{b} = \{ \hat{\beta}_{ik} \}$.

$$\text{SNR}^{-1} = \nu^{-1}$$ (17)

$$= \frac{E_b^{(m)}}{N_0} \gamma_{m0} r_{m0} g_m(r_{m0}) a_0^2 e^{\beta_{m0}} + \frac{1}{3G_m r_{m0} g_m(r_{m0}) a_0^2 e^{\beta_{m0}}}$$

$$= \frac{E_b^{(m)}}{N_0} \gamma_{m0} r_{m0} g_m(r_{m0}) a_0^2 e^{\beta_{m0}} + \frac{1}{3G_m r_{m0} g_m(r_{m0}) a_0^2 e^{\beta_{m0}}}$$

and

$$x = \sum_{k=1}^{K_m-1} a_0^2 e^{\beta_{ik}} r_{mk} g_m(r_{mk}) + \sum_{k=1}^{K_m-1} a_0^2 e^{\beta_{ik}} r_{mk} g_m(r_{mk})$$

$$= \sum_{i=0}^{S-1} K_{i-1} a_0^2 e^{\beta_{ik}} r_{mk} g_m(r_{mk})$$ (18)

The probability density function of $v$ is

$$f_v(v) = \frac{d}{dv} F_v(v)$$ (19)

where $F_v(v)$ can be described as following

$$F_v(v) = Pr(V < v) = \int \int F_v(v) f_B(\hat{b}) f_r(r) d\hat{b} dr$$

$$= 1 - \int_{-\infty}^{\infty} e^{-2 T^2 / \sigma_{\alpha}^2 / 2}$$

$$= \frac{1}{\sqrt{2\pi \sigma_{\alpha}^2}} \int_{-\infty}^{\infty} e^{-\frac{1}{2} \left( \frac{(v - \mu)^2}{\sigma^2} \right)} d\mu$$

The function $L_{\beta_{m0}, r_{m0}}$ is

$$L_{\beta_{m0}, r_{m0}} = \int_0^{\infty} \int_{-\infty}^{\infty} f_{\nu(r)} f_B(\beta) d\beta dr$$ (21)

**E. Convolutional Coding**

In order to obtain better performance, forward error correction codes is used in our multi-media multi-rate CDMA system. Suppose convolutional coding and Viterbi decoding are applied to the system, the average bit error probability at the decoder output is bounded [8] by

$$P_b \leq \frac{1}{2} \left[ T(D, N) + T(-D, N) \right]$$

$$+ \frac{1}{2} D T(D, N) - T(-D, N) \right]^{1/2}$$ (22)

In (22), $\frac{dP_v(D, N)}{d\nu}$ can be found in [8]. The coefficients of $\frac{dP_v(D, N)}{d\nu}$ for the best rate 1/2 and 1/3 with constraint length 14 have been obtained in [9]. To apply the upper bound to the multi-rate DS/CDMA systems, the term $\rho$ which represents the transition error probability of a binary symmetry channel (BSC) can be expressed by

$$\rho = p_{b}^{(i)}(r_{ik})$$ (23)

**III. OPTIMIZATION OF POWER CONTROL LAW**

In the previous sections, we derived the bit error rate for MS with arbitrary transmission rate and power control law. The BER will be the measure of Qos for every MS. Suppose the Qos of sub-system $i$ is $Q_i$, all qualified power control functions should meet the corresponding Qos requirement. Under these constraints, we would like to find the optimum power control functions so that the objectives can be achieved. In this paper, we propose two objective functions.

**Objective 1: Maximizing Capacity**

Maximum system capacity is the first objective. The function is the sum of the maximum allowable active users with given weight $\omega_i$ for sub-system $i$,

$$\text{max} J_i = \text{max} \left( \sum \omega_i K_i \right)$$ (24)

The constraints are,

$$P_{b}^{(i)}(r_{ik}) \leq Q_i, \forall i, k$$ (25)

$$0 \leq g_i(r_{ik}) \leq 1, \forall i, k$$ (26)

$$K_i \geq 1$$ (27)

**Objective 2: Minimize Power Consumption**
By given the number of users of each sub-system, we want to minimize the average transmission power of all MSs. The objective function is the sum of the average transmission power,

$$\min J_2 = \min \left\{ E[\sum g_i(r)] \right\} \quad (28)$$

The constraints are,

$$P_e^{(i)}(r_{ik}) \leq Q_i, \quad i, k \quad (29)$$

$$0 \leq g_i(r_{ik}) \leq 1, \quad i, k \quad (30)$$

From the BER equations presented in the last section, and the above objectives and constraints, it is obviously difficult to get a closed-form solution of optimal power control law. Therefore we resort to discrete optimizations. By dividing the distance $r \in [0, 1]$ to $N_r + 1$ equal spaced points, $0, \frac{1}{N_r}, \ldots, \frac{N_r - 1}{N_r}, 1$, we search the power control function at these points, $g_0[0], g_1[\frac{1}{N_r}], \ldots, g_{N_r}[\frac{N_r - 1}{N_r}], g_1[1]$. Each $g_i[m]$ is also a discrete function with value in 0, $\delta$, $2\delta$, $\ldots$, $N_r\delta = 1$. Suppose $N_r$ and $N_s$ are large enough, the discretized solutions is likely to approach to continuous power control law. Since the search space is very large, we use dynamic programming (DP) method to obtain the optimal (discrete) power control law.

IV. EQUAL BIT ENERGY POWER CONTROL LAW

Power control law that is proportional to the distance to BS has been used in CDMA system to maintain the same received bit-energy to background noise ratio ($E_b/N_0$) [3]. For multi-rate multi-media CDMA systems, equal received bit-energy strategy was proposed in [12, 13] where the receiver maintain equal received bit-energy for each mobile no matter what transmission rate it uses.

Considering the equation (15) and (18), to maintain equal received bit energy to noise ratio at a certain level $E_b/N_0$, such that

$$\frac{E_b'}{N_0} \gamma_{m0} r^{-\alpha} g_{m0}(r_{m0}) = \frac{E_b'}{N_0} \quad (31)$$

the following power control law for $k$th user with bit duration $T_i$ should be used

$$g_i(r_{ik}) = \gamma_{ik} r^{-\alpha} \quad (32)$$

V. NUMERICAL RESULTS

In this section, numerical results are presented to compare the optimal power control law and conventional power control law. We assume the propagation exponent is 2 and the variance of lognormal shadowing is 2 dB. Convolutional codes with coding rate 1/3 and constraint length 7 are used. For the reason of avoiding huge computation, Gaussian approximations method in section 2 is used to obtain the BER and dynamic programming is used to get optimal power control law. The performance of using enhanced Gaussian approximation can be found in [11].

Figure 1 shows the multiple access capability of single rate voice/data integrated system using equal bit energy power control law. When there are data users in the system, the total number of users can not exceed 10. However, the BER of voice users at this moment are $10^{-5}$ which is not necessary. The transmission power of voice users should be decreased to lower interference.

<table>
<thead>
<tr>
<th>Voice</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_0 = 256$</td>
<td>$G_1 = 128$</td>
</tr>
<tr>
<td>$Q_{\alpha 0} = 10^{-3}$</td>
<td>$Q_{\alpha 1} = 10^{-5}$</td>
</tr>
</tbody>
</table>

Tab. 1: System parameter for Figure 1 and 2.

The advantage of employing optimal power control law is shown in Figure 2. In figure 2, using optimal power control law can increase capacity for voice users when there are data users. In figure 3, we show that the capacity is gained by using optimal power control law in the multi-rate multi-media application. We assumed there are three types of service, voice, type-1 data, and type-2 data where each one has different parameters (Table 2).

<table>
<thead>
<tr>
<th>Voice</th>
<th>type-1 Data</th>
<th>type-2 Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_0 = 256$</td>
<td>$G_1 = 256$</td>
<td>$G_2 = 128$</td>
</tr>
<tr>
<td>$Q_{\alpha 0} = 10^{-3}$</td>
<td>$Q_{\alpha 1} = 10^{-5}$</td>
<td>$Q_{\alpha 2} = 10^{-5}$</td>
</tr>
</tbody>
</table>

Tab. 2: System parameter for Figure 3 and 4.

Given weights $w_i$ to each $K_i$, the objective function $J_1$ is obtained. In table 3 we compare the objective function $J_2$ in the system using optimal power control law and equal bit energy law. The optimal power control law for each type of service is shown in figure 4.

VI. CONCLUSIONS

In this paper we have proposed the optimal power control law for multi-media multi-rate CDMA systems. Since in the multi-media service different service has different QoS requirements, the mobiles only need to use the minimum required power (such that the BER is met) to transmit the signal. The power control law is not only dependent on the distance to BS but also the interference level (transmission rate and number of other users). Therefore the number of active multi-media users (capacity) can be increased and at the same time the power consumptions are minimized. The numerical results shows that the optimal power control law gives the best power allocation to multi-media users such that the objectives are achieved. In this paper, the analysis is focused on single cell and reverse link. The research on multiple cell optimal power control law will be conducted in the future. We also leave the implementation issues to the future.

REFERENCES


Fig. 1: Multiple access capability of a single-rate integrated voice/data CDMA system employing equal bit energy policy.

Fig. 2: System capacity of a single-rate integrated voice/data CDMA system employing optimal power control law and equal bit energy policy. $E_b/N_0 = 16.8$ dB.

Fig. 3: Objective function $J_1$ of a single-rate multi-media multi-rate CDMA system.

Fig. 4: Optimal power control law vs. equal bit energy law

<table>
<thead>
<tr>
<th>$(N_0, N_1, N_2)$</th>
<th>$(5,3,1)$</th>
<th>$(4,3,1)$</th>
<th>$(2,4,2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_2$</td>
<td>0.755730</td>
<td>0.719210</td>
<td>0.867720</td>
</tr>
<tr>
<td>$J_2$</td>
<td>N/A</td>
<td>1.699500</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Tab. 3: Average power consumption $J_2$. N/A denotes the BER $> Q_{av}$.